Average Run Length for Multivariate T^2 Control Chart Technique With Application

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Abstract:

A control chart is a tool that monitors quality characteristics of a process to insure that process control is being maintained when monitoring multiple characteristics that are correlated, it is imperative to use multivariate control chart. Hotelling's T^2 quality control chart is used to determine whether or not the process mean vector for two or more variables is in-control. It is allow us to simultaneously monitor whether two or more related variables are in control, and it is shown that multivariate quality control chart do not indicate which variables cause the out-of-control signal so that the interpretation of the out-of-control signal. This paper effort explores designing the multivariate T^2 quality control charts for process variability and methods that address which variable(s) caused the out-of-control signal. Industry fertilizers is important one of the chemical industries in Egypt, so that this work concerns the fertilizers industries quality control, especially urea fertilizer with application on Delta fertilizer and chemical industries which is considered on of the leading companies the field of fertilizer production in Middle east with application of multivariate quality control procedures to achieve best one procedure for multivariate quality control . This application shows that the company should use the multivariate quality control chart to determine whether or not the process is in – control because the production have several correlated variables, and the used of the separate control charts is misleading because the variables jointly affect the process. The used of separate univariate control charts in multivariate situation lead to a type I error and the probability of a point correctly plotting in-control are not equal to their expected values.

Key Words: Quality Control, Multivariate Analysis, Hotelling's T^2 .

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1. Introduction:

In today's highly competitive industrial environment, better control of chemical processes is an important step towards increasing the efficiency of production facilities. Improved process control could have a positive impact on chemical process in several ways: improved compliance with increasingly stringent environmental regulations, reduced generation of hazardous wastes and more consistent quality of the final product. A commonly used tool in statistical quality control of continuous production process is the control chart. Typically process monitoring applies to systems or processes in which only one variable is measured and tested. There are many processes which the simultaneous monitoring or control of two or more quality characteristics is necessary.

The most familiar multivariate process monitoring and control procedure is the Hotelling's T^2 control chart for monitoring the mean vector of the process. It is a direct analog of the univariate Shewhart $\bar{X}$ chart. Shewhart, a pioneer in the development of the statistical control chart (Shewhart charts), first recognized the need to consider quality control problems as multivariate in character. Hotelling (1947) did the original work in multivariate quality control. He applied his procedure, which assumed that P-quality characteristics are jointly distributed as P-variate normally and random samples of size *n* are collected across time from the process. T^2 is sensitive to shifts in the means, as well as to shifts in the variance, but it cannot distinguish between location shifts and scale shifts.

Multivariate charts are also useful for monitoring quality profiles as discussed by Woodall et al. (2004). Alt (1995) defined two phases in constructing multivariate control charts, with Phase I divided into two Stages. In the retrospective Stage 1 of Phase I, historical data (observations) are studied for determining whether the process was in control and to estimate the in-control parameters of the process. The Hotelling's T^2 control chart is utilized in this stage (Alt and Smith, 1998, Tracy et al. 1992, and Wieda,1994). In Phase II, control charts are used with future observations for detecting possible departure from the process parameters estimated in Phase I. In Phase II, one uses charts for detecting any departure from the parameter estimates, which are considered in the in-control process parameters (Vargas,2003).

An important aspect of the Hotelling's T^2 control chart is how to determine the sample variance-covariance matrix used in the calculation of the chart statistics, the upper control limit (*UCL*) and the lower control limit (*LCL*).

Onwuka (2012) discussed the Principal Component Analysis and Hotelling's T^2 tests were used, with the 3-characteristics measured showing negligible low correlation with nearly all the correlation coefficients small.

2. Multivariate Quality Control Chart:

It is shown that the multivariate quality control charts are powerful and simple visual tools for determining whether the multivariate process is in-control or out-of-control. In other words, control charts can help us to determine whether the process average (center) and process variability (spread) are operating at constant levels. Control charts help us focus problem – solving efforts by distinguishing between common and assignable cause variation. Multivariate control chart plot statistical from more than one related measurement variable. The multivariate control chart shows how several variables jointly influence a process or outcome.

It is demonstrated that if the data include correlated variables the use of separate control chart is misleading because the variables jointly affect the process. If we use separate univariate control chart in a multivariate situation, type I error and probability of a point correctly plotting in- control are not equal to their expected values the distortion of those values increases with the number of measurement variables.

It is shown that multivariate control chart has several advantages in comparison with multiply univariate charts:

- *The actual control region of the related variables is represented.*
- *We can maintain specification type I error.*
- *A signal control limit determines whether the process is in control.*
- *Multivariate control chart simultaneously monitor two or more correlated variables. To monitor more than one variable using univariate charts, we need to create a univariate charts for each variable.*
- *The scale on multivariate control charts unrelated to the scale of any of the variables.*
- *Out-of-control signals in multivariate charts do not reveal which variable or combination of variables cause the signal.*

A multivariate control chart consists of:

- *Plotted points, each for which represents a rational subgroup of data sampled from the process, such as a subgroup mean vector* *individual observation, or weighted statistic.*

- *A center line, which represents the expected value of the quality characteristics for all subgroups.*
- *Upper and lower control limits (UCL and LCL), which are set a distance above and below the center line. These control limits provide a visual display for the expected amount for variation. The control limits are based on the actual behavior of the process, not the desired behavior or specification limits. A process can be in control and yet not be capable of meeting requirements*.

3. Construction of Hotelling,s T^2 Control Chart:

The Hotelling multivariate control chart signals that a statistically significant shift in the mean has occurred as soon as:

$$\chi^2 = (\bar{X}_i - \mu_o)' \, \Sigma^{-1}(\bar{X}_i - \mu_o) \qquad (1)$$

If the sample covariance matrix Σ and the sample mean vector μ_0 are known, but if Σ and μ_0 are known, then the T^2 statistic is the appropriate statistic for the Hotelling multivariate control chart. In this case the sample covariance matrix, S and sample mean vector $\bar{X}$, are used to estimate Σ and μ_0 respectively.

This statistic has the from:

$$T^2 = (\bar{X}_i - \bar{\bar{X}})' \, S^{-1}(\bar{X}_i - \bar{\bar{X}}) \qquad (2)$$

Suppose that we have a random sample from a multivariate normal distribution – Say, X_1 , X_2 , X_3 , …, X_n where the i^{th} sample vector contains observations , X_{i1} , X_{i2} , X_{i3} , …, X_{ip} .

Let the sample mean vector is:

$$\underline{\bar{X}}_{1\times p} = (\bar{X}_1, \bar{X}_2, \dots, \bar{X}_p)$$

where

$$\bar{X}_i = \sum_{L=1}^{n} X_{iL} \quad , \qquad (i = 1,2, \dots, p)$$

and the sample covariance matrix is:

$$S = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1p} \\ S_{21} & S_{22} & \dots & S_{2p} \\ . & . & & . \\ . & . & & . \\ S_{p1} & S_{p2} & \dots & S_{pp} \end{bmatrix}$$

where $(ij)^{th}$element of S-matrix is the estimated covariance between the variables i and j

$$S_{ij} = \frac{1}{n-1} \sum_{L=1}^{n} (X_{iL} - \bar{X}_i)\,(X_{jL} - \bar{X}_j)$$

Not that we can show that the sample mean vector and the sample covariance matrix are unbiased estimators of the corresponding population quantities that is

$$E\left(\underline{\bar{X}}\right) = \underline{\mu} \quad and \quad E\,(S) = \Sigma \tag{3}$$

Seber (1981) gives the distribution properties of this estimate as follows:

(i) $\bar{X} \sim N_p\ (\mu, \frac{1}{n}\Sigma)$ (4)

(ii) If $\bar{X}$ distribution as in (i) then:

$$n\,(\bar{X} - \mu)^{'}\Sigma^{-1}(\bar{X} - \mu) \sim \chi^2_p \tag{5}$$

(iii) $(n-1)S \sim W_p\ (n-1, \Sigma)$

where $W\ (\ .\ ,\ .)$ stands for the Wishart distribution.

(iv) If Z and D are independent, random variables distributed respectively as:

$$Z \sim N_p(0, \Sigma_Z)$$
$$(n-1)D \sim W_p(n-1, \Sigma_Z)$$

then the quadratic form:

$$T^2 = Z^{'}\ D^{-1}\ Z \tag{6}$$

is distributed as:

$$T^2 \sim \frac{(n-1)p}{(n-p)}\ F_{p,n-p} \tag{7}$$

(v) If $Z \sim N_p(0, \Sigma_Z)$ and $(n-1)D \sim W_p(n-1, \Sigma_Z)$, $(n-1) > p$ where $(n-1)D$ can be decomposed as:

$$(n-1)D = (n-2)D_1 + ZZ'$$

where :

$$(n-2)D_1 \sim W_p(n-2, \Sigma_Z)$$

and Z is independent of D_1 then the quadratic form :

$$T^2 = Z' \ D^{-1} \ Z \qquad (8)$$

is distributed as:

$$T^2 \sim (n-1)\beta \ (p, n-p-1)$$

where :

$\beta \ (p, n-p-1)$ is the central Beta distribution.

(vi) If the sample is composed of k subgroups of size n with subgroup means $\bar{X}_j, j = 1,2,3, \dots, k$ and grand mean $\bar{\bar{X}}, \ i.e.$

$$\bar{X} = \sum_{j=1}^{k} \bar{X}_j \ /k = \sum_{j=1}^{k} \sum_{i=1}^{n} X_{ij} \ /kn$$

then

$$\sqrt{\frac{kn}{k-1}} \left(\bar{X}_j - \bar{\bar{X}}\right) \sim N_p \ (0, \Sigma) \qquad (9)$$

(vii) If the sample is composed of K subgroups of n identically distributed multivariate normal observations and if S_j is the sample covariance matrix from the j^{th}subgroup, j = 1, 2 , 3, …, K) then :

$$\Sigma \ (n-1)S_j \sim W_p \ (K_{n-1}, \Sigma) \qquad (10)$$

these distributional properties of $\bar{X}, \ S \ and \ T^2$ are used in the multivariate quality control procedures.

Now, we present two versions of Hotelling T^2 chart:

a) Subgroup Data:

Suppose that P-related quality characteristic $X_1, X_2, X_3, \dots, X_p$ are controlled jointly according to the P-multivariate normal distribution. The procedure requires computing the sample mean for each of the P-quality characteristics from a sample of size n.

Let the set of quality characteristic means is represented by the $(p \times 1)$ vector $\bar{X}$ as:

$$\bar{X} = \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \vdots \\ \bar{X}_p \end{bmatrix}$$

Then the test statistic plotted on the Chi-square control chart for each sample is:

$$\chi_0^2 = n\left(\underline{\bar{X}} - \underline{\mu}\right)' \Sigma^{-1}\left(\underline{\bar{X}} - \underline{\mu}\right) \qquad (11)$$

where :
$\underline{\mu}' = (\mu_1, \mu_2, \dots, \mu_p)$ is the $(p \times 1)$ vector of in-control means for each quality characteristic and Σ is covariance matrix.

Now, suppose that m-subgroup are available. The sample means and variances are calculated from each subgroup as usual that is:

$$\bar{X}_{jk} = \frac{1}{n}\sum_{i=1}^{n} X_{ijk}$$

$$S_{jk}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_{ijk} - \bar{X}_{jk}\right)^2,$$

$$j = 1, 2, \dots, p \;; k = 1, 2, \dots, m \qquad (12)$$

Where X_{ijk} is the i^{th} observation on the j^{th} quality characteristic in the k^{th} subgroup.

$$S_{jhk} = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_{ijk} - \bar{X}_{jk}\right)(X_{ihk} - \bar{X}_{hk}),$$

$$k = 1, 2, \dots, n\,, j \neq h \qquad (13)$$

Represents the covariance between quality characteristic j and quality characteristic h in the k^{th} subgroup.

The statistics $\bar{X}_{jk}$, S_{jk}^2, S_{jhk} are the averaged over all m-subgroups to obtain

$$\bar{\bar{X}}_j = \frac{1}{m}\sum_{k=1}^{m} \bar{X}_{jk} \;, \qquad j = 1,2, \dots, p$$

$$\bar{S}_j^2 = \frac{1}{m}\sum_{k=1}^{m} S_{jk}^2 \quad , \qquad j = 1,2,\dots,p$$

and

$$\bar{S}_{jh} = \frac{1}{m}\sum_{k=1}^{m} S_{jhk} \qquad (14)$$

where $j \neq h\ and\ \bar{\bar{X}}_j$ are the i^{th} elements of the $(p \times 1)$ sample mean vector $\underline{\bar{\bar{X}}}\ and\ (p \times p)$ average of sample covariance matrices S is formed as:

$$S = \begin{bmatrix} \bar{S}_1^2 & \bar{S}_{12} & \dots & \bar{S}_{1p} \\ & \bar{S}_2^2 & \dots & \bar{S}_{2p} \\ & & \dots & \\ & & & \bar{S}_p^2 \end{bmatrix} \qquad (15)$$

There are consider the unbiased estimate of $\mu\ and\ \Sigma$ when the process is in control.

If, we replace μ with $\bar{\bar{X}}\ and\ \Sigma$ with S in (11), the test statistic now becomes

$$T^2 = n\left(\bar{X} - \bar{\bar{X}}\right)' S^{-1}\left(\bar{X} - \bar{\bar{X}}\right) \qquad (16)$$

Alt (1985) has pointed out that there are two distinct phases of control chart using.

Phase (I) is the use of the chart for establishing control, that is, testing whether the process was in control when m subgroups were drawn and the sample statistic $\bar{\bar{X}}$ and S computed. The objective in phase (I) is to obtain an in-control set of observations, so that control limits can be established for phase (II) which is the monitoring of future production.

In the phase (I) the control limits for the T^2-control chart are given by:

$$\left.\begin{aligned} UCL &= \frac{P(m-1)(n-1)}{mn-m-p+1} F_{\alpha,P,mn-m-p+1} \\ LCL &= 0 \end{aligned}\right\} \qquad (17)$$

In the phase (II) when the chart is used for monitoring future production, the control limits are as follows:

$$\left.\begin{array}{l} UCL = \frac{P(m+1)(n-1)}{mn-m-p+1} F_{\alpha,P,mn-m-p+1} \\ LCL = 0 \end{array}\right\} \quad (18)$$

When the parameters μ and Σ are estimated from a large number of subgroups, it is often to use $UCL = \chi^2_{\alpha,p}$ as the upper limit in both phases. Retrospective analysis of samples to test for statistical control and establish control limits also occurs in the univariate control chart setting. For the $\bar{X}$-chart, it is well-known that if use $m \geq 20$ *or* 25 , samples, the distribution between phase I and phase II limits is usually unnecessary, because the phase I and phase II limits will nearly coincide. However, with multivariate control charts, we must be careful.

Lowry and Montgomery (1995) showed that in many situations a large number of samples would be required before the exact phase II control limits are well approximate by the Chi-square.

b) Individual Observations:

In some situation the subgroup size is naturally $n = 1$. Suppose that m samples each of size $n = 1$ are available and that p is the number of quality characteristics observed in each sample. The Hotelling T^2 statistic becomes:

$$T^2 = (X - \bar{X})' S^{-1} (X - \bar{X}) \quad (19)$$

Ryan (1989) defined the phase II control limits for this statistic as:

$$\left.\begin{array}{l} UCL = \frac{P(m+1)(m-1)}{m(m-p)} F_{\alpha,P,m-p} \\ LCL = 0 \end{array}\right\} \quad (20)$$

Jackson (1985) suggested that for large m $(m > 100)$ then we can use an approximate control limit, either

$$UCL = \frac{P(m-1)}{(m-p)} F_{\alpha,P,m-p} \quad (21)$$

Or

$$UCL = \chi^2_{\alpha,p} \quad (22)$$

Equation (22) is only appropriate if the covariance matrix is known.

Lowry and Montgomery (1995) suggested that if p is large-say $p \geq 10$ then at least 250 samples must be taken $(m \geq 250)$ before Chi-square upper control limit is a reasonable approximation to the correct value.

Tracy Young and Mason (1992) point out that if n = 1, the phase (I) limits should be based on a beta distribution that is, the phase (I) limits defined as:

$$\left.\begin{aligned} UCL &= \frac{(m-1)^2}{m}\beta_{\alpha,\frac{P}{2},\frac{m-p-1}{2}} \\ LCL &= 0 \end{aligned}\right\} \quad (23)$$

4. Average Run Length (*ARL*) Performance:

Mason and Young (2002) suggested that the average run length (*ARL*) for a control procedure is defined as:

$$ARL = \frac{1}{P}$$

where *P* represents the probability of being outside the control region. For a process that is in-control, this probability is equal to α, the probability of type I error.

The *ARL* has a number of uses in both univariate and multivariate control procedures. They suggested that it can be used to calculate the number of observations that one would expect to observe, on average, before a false alarm occurs, this given by:

$$ARL = \frac{1}{\alpha}$$

Another use of the *ARL* is to compute the number of observations one would expect to observe before detecting a given shift in the process.
The probability of detecting the shift equals $(1-\beta)$, where β is the probability of a type II error. The ARL for detecting the shift is given by:

$$ARL = \frac{1}{1-\beta}$$

Multivariate control charts using Hotelling's T^2 statistic are popular and easy to use. A major advantage of Hotelling's T^2 statistic is that it can be shown to be the optimal test statistic for detecting a general shift in the process mean vector for an individual multivariate observation. However, the technique has several practical drawbacks. A major drawback is that when the T^2 statistic indicates that a process is out of control, it does not provide information in which variable or set variables is out of control. Further, it is difficult to distinguish location shift from scale shifts since the T^2 statistic is sensitive to both types of process changes.

5. The Application:

Delta Fertilizers and Chemical Industries is considered on of the leading companies in the field of fertilizers production in Egypt. About 4500 employees are working for it, on the various managerial levels. Urea production is one of the major products of the company. The production of urea occurs through three stages, summarized as follows:

5.1 High Pressure stage:

In this stage, urea is produced through two reactions; the first reaction occurs by condensation of Ammonia Gas and Carbon dioxide under high pressure and temperature for the sake of the production of intermediate material, known as Carbamate .The second reaction happens by separating the water from the Carbamate in order to a chive urea. In this, stage the condensation of urea approximately 56%.

It contains 16 variables, these are:

X1	E-201Outlet Temperature
X2	Outlet Cold NH3 from E- 201
X3	Co2 to Train
X4	Co2 pressure to Synthesis
X5	Co2 after E-22
X6	R-201
X7	Temperature in Reactor R-201
X8	Temperature in Reactor R-201
X9	Temperature in Reactor R-201
X10	Temperature in Reactor R-201
X11	Stripper level
X12	Liquid Leaving the Stripper
X13	Stream from E-204 to j-201
X14	Conditioned water to Scrubber E-204
X15	Conditioned water from Scrubber E-204
X16	Stream from j-203

Table analysis of laboratory in this stage:

$t_{1.1}$	NH_3	Rector outlet

$t_{1.2}$	CO_2	Rector outlet
$t_{1.3}$	UR	Rector outlet
$t_{1.4}$	B_1	Rector outlet
$t_{1.5}$	H_2O	Rector outlet
$t_{2.1}$	NH_3	Stripper outlet
$t_{2.2}$	CO_2	Stripper outlet
$t_{2.3}$	UR	Stripper outlet
$t_{2.4}$	B_1	Stripper outlet
$t_{2.5}$	H_2O	Stripper outlet

5.2 Low Pressure Stage:

In this stage, the condensation of urea liquid rises from 56% to 71%. This happens through the decomposition of the remaining Carbamate and the elimination of water under low pressure.

It contains seven variables, these are:

y1	Urea Solution from stripper E-202
y2	Steam to E-205
y3	Urea Carbonate Solution from stripper T-201 to E-205
y4	Gas leaving T-201
y5	Level in TK-201
y6	P-203
y7	Urea Solution in TK-201

Table analysis of laboratory in this stage:

$t_{3.1}$	NH_3	D 202 Outlet
$t_{3.2}$	CO_2	D 202 Outlet
$t_{3\cdot3}$	UR	D 202 Outlet
$t_{3.4}$	B_1	D 202 Outlet
$t_{3.5}$	H_2O	D 202 Outlet
$t_{4.1}$	NH_3	In TK 201
$t_{4.2}$	CO_2	In TK 201
$t_{4.3}$	UR	In TK 201
$t_{4.4}$	B_1	In TK 201
$t_{4.5}$	H_2O	In TK 201
$t_{5.1}$	NH_3	In PI 302
$t_{5.2}$	CO_2	In PI 302
$t_{5.3}$	UR	In PI 302

5.3 Evaporation and Prilling stage:

This stage occurs by two stage:

- *Firstly, Evaporation stage:*

In this stage, the condensation of urea rises from 71% to 98.7% approximately and the urea liquid trams forms to urea melt. This happens under high pressure and temperature.

- *Secondly, Prilling Stage:*

In this stage, the urea melt is through formed into prilling in the prilling tower.

It contains four variables, these are:

Z_1	Urea Solution from D-204 to E-209
Z_2	D- 205 Vacuum
Z_3	Urea to periling tower X-202
Z_4	E- 211 Vacuum

Table analysis of laboratory in this stage:

$t_{6.1}$	B_1
$t_{6.2}$	H_2O
$t_{6.3}$	Pills > 3.35
$t_{6.4}$	Pills 3.35: 2.4
$t_{6.5}$	Pills 2.4 : 1.4
$t_{6.6}$	Pills 1.4 :1.0
$t_{6.7}$	Pills < 1.0
$t_{6.8}$	UR

5.4 Data description:

For the application of multivariate quality control, chart data originate from urea production process, which consists of the three stages and the analysis of laboratory, which discussed above.

The number of the sample is 732 observations taken per hour.
The advantages of this sample that, it has several variables and several stage of the production. This advantage of the production is the basic

reason for choosing this production to allow us to study the multivariate quality control charts.

In this application, we shall introduce the most common using technique of multivariate quality control chart; Hotelling T^2 chart.

A Hotelling T^2 chart consists of:

- *Plotted points, each of which represents T^2 statistic for each observation.*
- *A center line (green), which is the median of the theoretical distribution of T^2 statistic.*
- *Control limits (red), which provide a visual means for assessing whether the process is in-control. The control limits represent the expected variation.*

MINITAB marks points outside of the control limits with a red symbol. MINITAB indicates which points is out-of-control by using decomposition of T^2 statistic, along with the P-value for each significant variable.

(1) T squared chart of X1,..., X16 and $t_{1.1}$, ..., $t_{2.5}$

Test Results for T squared Chart of X1,..., X16 and $t_{1.1}$, ..., $t_{2.5}$

TEST. One point beyond control limits.

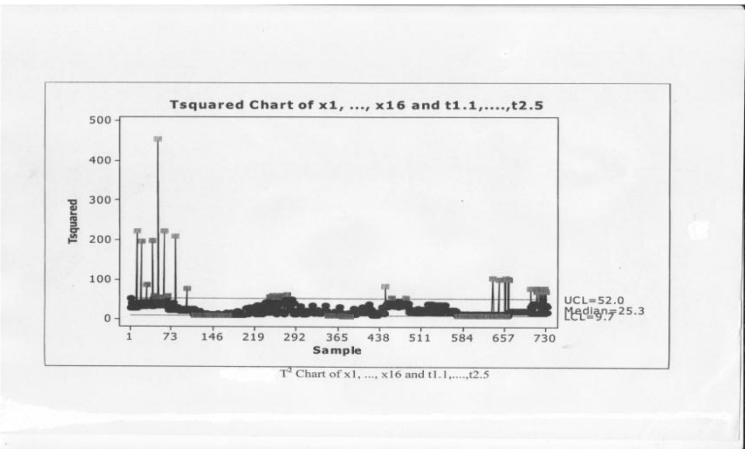
Test Failed at points: (Less Than *LCL*)

114	118	123	128	133	134	138	140	143	147
151	156	161	166	167	171	173	349	352	353
354	361	373	374	378	384	385	386	576	578
582	586	590	594	598	600	604	608	612	616
620	624	626	630	634	638	642	646	650	652
656	660								

Test Failed at points: (Greater Than *UCL*)

13	20	30	40	43	45	48	50	52	56
60	66	80	100	245	247	250	252	254	259
261	265	276	448	458	483	636	647	657	658
663	664	703	714	715	718	721	724	727	730

Fig. (1)



We Can Summarize the T^2 chart of X1,…, X16 and $t_{1.1}$, …, $t_{2.5}$ as follows:

- The lower and upper control limits are 9.7 and 52, respectively. Therefore, we expect the T^2 Statistics to fall between 9.7 and 52.The center line or median, is 25.3.
- Test results indicate that 52 point less than *LCL*, for example, point 114 exceeds the lower control limit.
- Test results indicate that 40 points greater than *UCL*, for example, the test results indicate that point13 exceeds the upper control limit.
- Test results indicate 92 Point through beyond the control limits. Then the out-of-control rate 12.6% and the in-control rate 87.4%.

(2) T squared chart of y1,…, y7 and $t_{3.1}$, …, $t_{4.5}$

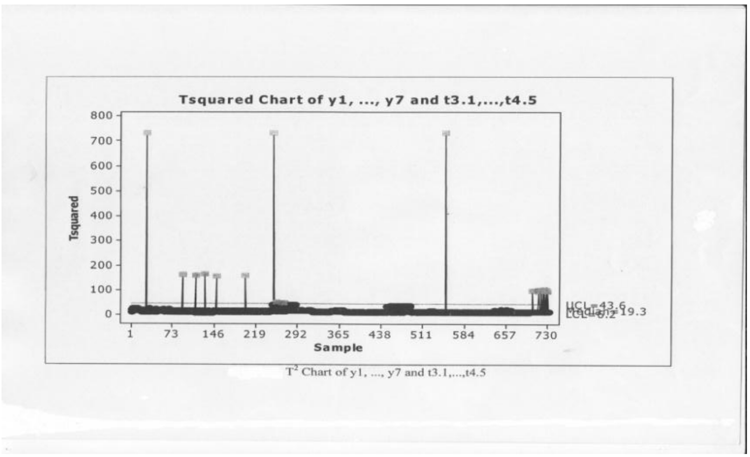
Test Results for T squared Chart of y1,…, y7 and $t_{3.1}$, …, $t_{4.5}$

TEST. One point beyond control limits.

Test Failed at points: (Greater Than *UCL*)

28	91	114	130	150	200	250	256	264	268
551	703	714	715	718	721	724	727	730	

Fig. (2)



We Can summarized the T^2 chart of y1,…, y7 and $t_{3.1}$, …, $t_{4.5}$ as follows:

- The lower and upper control limits are 0.2 and 43.6, respectively. Therefore, we expect the T^2statistics to fall between 0.2 and 43.6. The center line, or median, is 19.3.
- Test results indicate that 19 Point greater than *UCL*, for example test results indicate that Point 91 exceeds the upper control limit.
- Test results indicate 19 Point that are beyond the control limit. Then the out of control rate 2.59% and the in-control rate 97.41%.

(3) T squared chart of Z_1,…, $t_{6.8}$

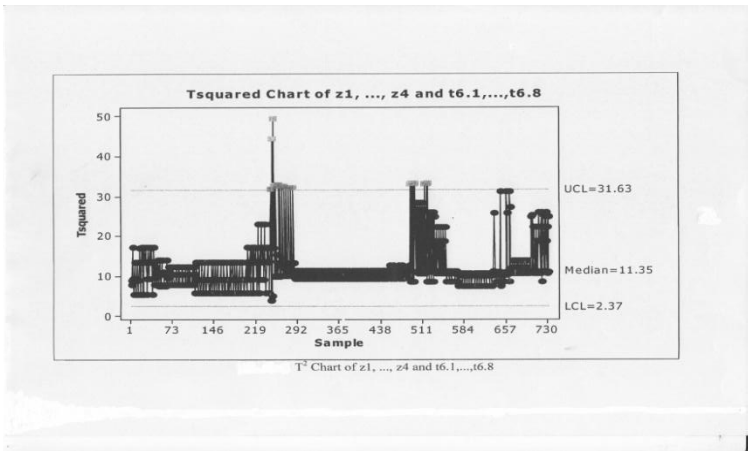
Test Results for T squared Chart of Z_1,…, $t_{6.8}$

TEST. One point beyond control limits.

Test Failed at points: (Greater Than *UCL*)

245	248	250	252	254	259	261	265	269	272
276	280	284	489	491	493	495	497	515	517
519									

Fig. (3)



We Can Summarized the T^2 chart of $Z_1,\ldots, t_{6.8}$ as follows:

- The lower and upper control limits are 2.37 and 31.63, respectively. Therefore, we expect the T^2statistics to fall between 2.37 and 31.63. The center line, or median, is 11.35.
- Test results indicate that 21 Point greater than *UCL*, for example test results indicate that Point 245 exceeds the upper control limit.
- Test results indicate 21 Point that are beyond the control limit. Then the out of control rate 2.87% and the in-control rate 97.13%.

6. Results and Conclusions:

Hotelling's T^2 charts used to determine whether or not the process mean vector (A vector of the process means that accounts for the mean of each charted variable) for two or more variables is in-control.

An in-control process exhibits only random variation with the control limits.

An out-of-control process exhibits unusually variation, which may be due to the process of assignable causes (unusual occurrences that are not normally part of the process).

T^2 charts allow us to simultaneously monitor whether two or more related variables are in control.

6.1 Test Results of the Application:

The application is shown that in High process stage, test results of T^2 chart indicate that the out-of-control percentage 87.4% and the in-control percentage 12.6%, and it shown that in Low process stage, test results of T^2 chart indicate that the out-of-control percentage 2.59% and the in-control percentage 97.41%.

It is shown that in the Evaporation and Prilling stage, test results of T^2 chart indicates that the out-of-control percentage 2.87% and the in-control percentage 97.13%.

6.2 Finally:

The company should use multivariate Hotelling's T^2 quality control chart to monitor the quality of the urea production.

Too, the company should use the Hotelling's T^2 chart to determine variables which causes the out-of-control signals.

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